Velocity Motion Model (cont)

Velocity Motion Model

- center of circle

$$\begin{pmatrix} x^* \\ y^* \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} -\lambda \sin\theta \\ \lambda \cos\theta \end{pmatrix} = \begin{pmatrix} \frac{x+x'}{2} + \mu(y - y') \\ \frac{y+y'}{2} + \mu(x' - x) \end{pmatrix}$$

where

$$\mu = \frac{1}{2} \frac{(x - x')\cos\theta + (y - y')\sin\theta}{(y - y')\cos\theta - (x - x')\sin\theta}$$

Velocity Motion Model

Algorithm motion_model_velocity(x_t, u_t, x_{t-1})**:**

$$\mu = \frac{1}{2}\;\frac{(x-x')\cos\theta + (y-y')\sin\theta}{(y-y')\cos\theta - (x-x')\sin\theta}$$

$$x^* = \frac{x+x'}{2} + \mu(y-y')$$

$$y^* = \frac{y+y'}{2} + \mu(x'-x)$$

$$r^* = \sqrt{(x-x^*)^2 + (y-y^*)^2}$$

$$\Delta\theta = \operatorname{atan2}(y'-y^*, x'-x^*) - \operatorname{atan2}(y-y^*, x-x^*)$$

$$\hat{v} = \frac{\Delta\theta}{\Delta t}\, r^*$$

$$\hat{\omega} = \frac{\Delta\theta}{\Delta t}$$

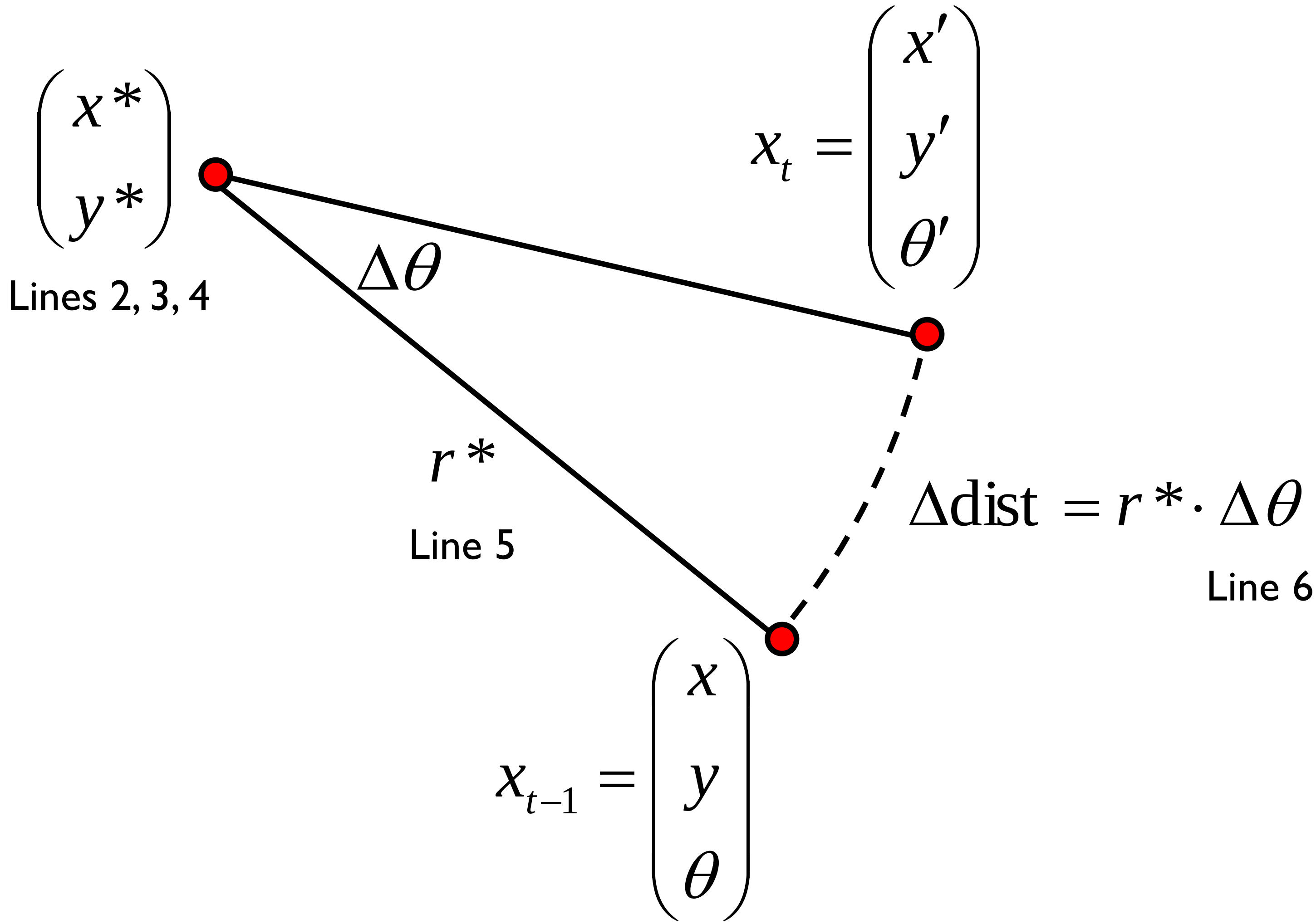
$$\hat{\gamma} = \frac{\theta'-\theta}{\Delta t} - \hat{\omega}$$

return $\mathbf{prob}(v-\hat{v}, \alpha_1\, v^2 + \alpha_2\, \omega^2) \;\cdot\; \mathbf{prob}(\omega-\hat{\omega}, \alpha_3\, v^2 + \alpha_4\, \omega^2)$

$\cdot\; \mathbf{prob}(\hat{\gamma}, \alpha_5\, v^2 + \alpha_6\, \omega^2)$

Velocity Motion Model

- rotation of $\Delta\theta$ about (x^*, y^*) from (x, y) to (x', y') in time Δt



Velocity Motion Model

- given $\Delta\theta$ and Δdist we can compute the velocities needed to generate the motion

$$\hat{u}_t = \begin{pmatrix} \hat{v}_t \\ \hat{\omega}_t \end{pmatrix} = \begin{pmatrix} \Delta\text{dist} \;/\; \Delta t \\ \Delta\theta \;/\; \Delta t \end{pmatrix}$$

Steps 7, 8

- notice what the algorithm has done
  - it has used an inverse motion model to compute the control vector that would be needed to produce the motion from $x_{t\text{-}1}$ to x_t
  - in general, the computed control vector will be different from the actual control vector u_t

Velocity Motion Model

- recall that we want the posterior conditional density

$$p(x_t \mid u_t, x_{t-1})$$

  of the control action u_t carrying the robot from pose x_{t-1} to x_t in time Δt
- so far the algorithm has computed the required control action $\hat{u}_t$ needed to carry the robot from position $(x\ y)$ to position $(x'\ y')$
  - the control action has been computed assuming the robot moves on a circular arc

Velocity Motion Model

- the computed heading of the robot is $\hat{\theta} = \theta + \Delta\theta$
- the heading should be θ'
- the difference is

$$\begin{aligned}\theta_{\text{err}} &= \theta' - \hat{\theta} \\ &= \theta' - \theta - \Delta\theta\end{aligned}$$

- or expressed as an angular velocity

$$\begin{aligned}\gamma_{\text{err}} &= \frac{\theta_{\text{err}}}{\Delta t} \\ &= \frac{\theta' - \theta}{\Delta t} - \hat{\omega}\end{aligned}$$

Line 9,
Eq 5.25, 5.28

Velocity Motion Model

- similarly, we can compute the errors of the computed linear and rotational velocities

$$
\begin{aligned}
v_{\text{err}} &= v - \hat{v} \\
&= \frac{\Delta \text{dist}}{\Delta t}
\end{aligned}
$$

$$
\begin{aligned}
\omega_{\text{err}} &= \omega - \hat{\omega} \\
&= \frac{\Delta \theta}{\Delta t}
\end{aligned}
$$

Velocity Motion Model

- if we assume that the robot has independent control over its controlled linear and angular velocities then the joint density of the errors is

$$p(v_{\text{err}}, \omega_{\text{err}}, \gamma_{\text{err}}) = p(v_{\text{err}})\ p(\omega_{\text{err}})\ p(\gamma_{\text{err}})$$

- what do the individual densities look like?

Velocity Motion Model

- the most common noise model is additive zero-mean noise, i.e.

$$\begin{pmatrix} \hat{v} \\ \hat{\omega} \end{pmatrix} = \begin{pmatrix} v \\ \omega \end{pmatrix} + \begin{pmatrix} v_{\text{noise}} \\ \omega_{\text{noise}} \end{pmatrix}$$

actual velocity	commanded velocity	noise

- we need to decide on other characteristics of the noises
  - "spread" variance
  - "skew" skew
  - "peakedness" kurtosis
- typically, only the variance is specified
  - the true variance is typically unknown

Velocity Motion Model

- the textbook assumes that the variances can be modeled as

$$\mathrm{var}(v_{\mathrm{noise}}) = \alpha_1 v^2 + \alpha_2 \omega^2$$
$$\mathrm{var}(\omega_{\mathrm{noise}}) = \alpha_3 v^2 + \alpha_4 \omega^2$$

Eq 5.10

where the α_i are robot specific error parameters

- the less accurate the robot the larger the α_i

Velocity Motion Model

- a robot travelling on a circular arc has no independent control over its heading
  - the heading must be tangent to the arc

$$\theta' = \theta + \hat{\omega}\,\Delta t$$

- this is problematic if you have a noisy commanded angular velocity ω
- thus, we assume that the final heading is actually given by

$$\theta' = \theta + \hat{\omega}\,\Delta t + \hat{\gamma}\,\Delta t \qquad \text{Eq 5.14}$$

where $\hat{\gamma}$ is the angular velocity of the robot spinning in place

Velocity Motion Model

- the book assumes that

$$\hat{\gamma} = 0 + \gamma_{\text{noise}}$$

actual velocity — noise

where

$$\text{var}(\gamma_{\text{noise}}) = \alpha_5 v^2 + \alpha_6 \omega^2 \qquad \text{Eq 5.15}$$